

# Modeling and Controller Design for the Air-to-Air Missile Uncertain System

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## Abstract

The guidance and control problem of the air-to-air missile system is studied. A nonlinear, coupling dynamic model of the air-to-air missile with six degrees of freedom is investigated, and a uncertain control system is proposed according to some assumptions and simplifications. Then, based on Lyapunov stability theory, a Lyapunov function is employed, and a controller is designed for the air-to-air missile. Numerical simulations show that the control system proves the correctness and has preferably tracking performance and illustrate the effectiveness of the proposed controller.

**Keywords:** uncertain system; dynamic model; controller design; air-to-air missile system.

## 1. Introduction

In the future air combat, the missile must have strong maneuverability and off-axis emissive capability and all range attack ability, especially rear hemisphere attacking target ability. Therefore, the air-to-air missile in the future should have huge off-axis capability and the better maneuverability and agility.

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The common methods in the integrated design of missile guidance and control include sliding mode control, backstepping design, feedback linearization and robust parameterization. But it is difficult to get analytical form of the control law using the sliding mode control method for maneuver target and unstable aerodynamic. The complexity of the control algorithm will increase with the order number increase sharply using backstepping design. Feedback linearization method needs accurate object model. Robust parameterization approach must establish a complete parametric system meeting the performance index of control law. Therefore, these methods are limited in practical application. Especially for the case of arbitrary maneuvering target and gust disturbance, the system is nonlinear, coupled and uncertain. The above methods are difficult to meet the system requirements. In recent years, due to the fast response, insensitivity to system parameters and external disturbances, simple algorithm and strong robustness, the optimal control method has attracted much attention in the integrated design of guidance and control. In recent years, due to the fast response, insensitivity to system parameters and external disturbances, algorithm simpleness and strong robustness, Lyapunov direct method has attracted much attention in the integrated design of guidance and control. Shtessel and his colleagues studied the integrated guidance and control problem based on first order, two order and higher order sliding mode control [1-3]. Shima and Idan and his colleagues [4-6] established the direct connection between the control input and the control target with the zero miss distance as the sliding surface, and design the integrated controller. Nathan and his colleagues [7-8] took the predicted collision point error as the sliding mode surface, and use the finite time convergence of the sliding mode state to meet the required constraints, and propose an integrated sliding mode control method. In this paper, the guidance and control problem of the air-to-air missile system is studied. A nonlinear, coupling dynamic model of the air-to-air missile with six degrees of freedom is investigated, and an uncertain control system is proposed according to some assumptions and simplifications. Then, based on Lyapunov stability theory, a Lyapunov function is employed, and a controller is designed for the air-to-air missile. The numerical simulation demonstrated the control effect of the proposed controller.

## **2. Dynamical System Modeling**

The research object of this paper is the air-to-air missile that uses aerodynamic layout with aerodynamic force / thrust vector composite control system, and in the active phase can use thrust vector control rudder or air rudder. Because the aerodynamic configuration of the missile uses normal pneumatic layout, so the aerodynamic characteristics of the missile rudder is simple. Considering this characteristic, the aerodynamic contribution of the air rudder is simply considered as a linear function of the rudder deflection angle. In addition, the influence of aerodynamic torsion angle on aerodynamic forces can be neglected without considering the aerodynamic coupling characteristics at high angles of attack. Based on the above considerations, in order to make the motion equation of the missile with six degrees of freedom not too complex, the following assumptions should be made:

- 1) The elastic mode of the missile is neglected, and the missile is considered as a rigid body;
- 2) Assuming that the thrust force of the engine is constant, and the force provided by the thrust vector deflection is only involved in the longitudinal and lateral motions of the missile. Thrust vector control actuator model assuming it thrust size is constant and assuming that only in the pitch and yaw plane surface deflection of thrust vector, and use two kinds of actuator to complete the two deflection (not from the rolling control system);

- 3) Ignoring the influence of gravity, considering only the action force and thrust force of the control force and thrust vector, and the influence of gravity can easily be compensated;
- 4) Due to the short time of compound control turn, the missile is considered to be in short period motion, and the missile, rotation inertia and velocity are considered constant;
- 5) The center of mass of missile remains unchanged.

The aerodynamic force / thrust vector composite control system of the missile is as follows:

$$\begin{aligned}
 \dot{\alpha} &= \omega_z - \omega_x \tan \beta \cos \alpha + \omega_y \tan \beta \sin \alpha - \frac{qS(C_n^\alpha \alpha + C_n^{\delta_z} \delta_z) \cos \alpha}{mV \cos \beta} \\
 &\quad + \frac{qS \sin \alpha}{mV \cos \beta} C_x - \frac{P \sin \alpha}{mV \cos \beta} + \frac{P \delta_{p_z} \cos \alpha}{mV \cos \beta} \\
 \dot{\beta} &= \omega_x \sin \alpha + \omega_y \cos \alpha + \frac{qS \cos \alpha \sin \beta}{mV} C_x - \frac{P \cos \alpha \sin \beta}{mV} + \\
 &\quad \frac{qS(C_n^\alpha \alpha + C_n^{\delta_z} \delta_z) \sin \alpha \sin \beta}{mV} + \frac{qS(C_z^\beta \beta + C_z^{\delta_\gamma} \delta_\gamma) \cos \alpha}{mV} + \\
 &\quad \frac{P \delta_{p_z} \sin \alpha \sin \beta}{mV} + \frac{P \delta_{p_\gamma} \cos \alpha}{mV} \\
 \dot{\gamma} &= \omega_x \\
 \dot{\omega}_x &= \frac{(m_x^{\bar{\omega}_x} \frac{\omega_x L}{V} + m_x^{\delta_x} \delta_x) qSL}{J_x} \\
 \dot{\omega}_\gamma &= \frac{m_\gamma^{\bar{\omega}_\gamma} \omega_\gamma L}{V J_\gamma} + \frac{(m_\gamma^{\delta_\gamma} \delta_\gamma + m_\gamma^\beta \beta) qSL}{J_\gamma} + \frac{(J_z - J_x) \omega_z \omega_x - l P \delta_{T_\gamma}}{J_\gamma} \\
 \dot{\omega}_z &= \frac{m_z^{\bar{\omega}_z} \omega_z L}{V J_z} + \frac{(m_z^{\delta_z} \delta_z + m_z^\alpha \alpha) qSL}{J_z} + \frac{(J_x - J_\gamma) \omega_\gamma \omega_x - l P \delta_{T_z}}{J_z} \tag{1}
 \end{aligned}$$

in which,  $\alpha, \beta, \gamma$  are attack angle, sideslip angle and roll angle, respectively; and  $\omega_x, \omega_\gamma, \omega_z$  are relevant rotational angular rates respectively;  $q$  is dynamic head;  $S$  is characteristic area,  $L$  is characteristic length,  $P$  is engine thrust force,  $m$  is missile mass,  $V$  is missile velocity,  $J_i$  ( $i = x, \gamma, z$ ) are projections of the missile inertia tensor on  $x, \gamma, z$  axes,  $\delta_i$  ( $i = x, \gamma, z$ ) are relevant rudder deflection angles,  $C_i^j$  are relevant aerodynamic coefficients,  $m_i^j$  are relevant aerodynamic moments.

It can be seen from (1) that the air-to-air missile control system model with six degrees of freedom has complex nonlinear and coupling characteristics. When considering gust disturbance, it also involves uncertainties, which makes the design of control system more difficult. In practical design, the model needs to be simplified. The usual practice is to assume that the missile does not roll during the target pursuit, and the three-dimensional

interception problem is decomposed into two dimensional interception problems in the longitudinal and lateral planes.

Based on the assumptions following:

- 1) The missile body does not roll;
- 2) The angle of attack and the sideslip angle of the missile are smaller;
- 3) The coupling term for the rest of the channel is bounded and unknown.

The approximate linear model is proposed in [9] as follows:

$$\begin{aligned}\ddot{\varepsilon} &= -\frac{2\dot{R}}{R}\dot{\varepsilon} - \frac{57.3qSC_\gamma^\alpha + P}{mR} + \Delta_{\varepsilon}, \\ \dot{\alpha} &= -\frac{57.3qSC_\gamma^\alpha + P}{mV}\alpha + \omega_z + \Delta_{\alpha}, \\ \dot{\omega}_z &= \frac{57.3qSLm_z^\alpha}{J_z}\alpha + \frac{qSL^2m_z^{\bar{\omega}_z}}{VJ_z}\omega_z + \frac{57.3qSLm_z^{\delta_z}}{J_z}\delta_z + \Delta_{\omega_z},\end{aligned}\tag{2}$$

in which,  $\varepsilon$  denotes the dip angle at the very moment.

Choose state variables for the air-to-air missile control system (2) in the following:

$$\begin{aligned}x &= [\dot{\varepsilon}, \alpha, \omega_z]^T \\ u &= \delta_z \\ y &= \dot{\varepsilon} = x_1\end{aligned}\tag{3}$$

and the system (2) is rewritten in the state-space representation:

$$\begin{aligned}\dot{x} &= \begin{bmatrix} a_{11} & a_{12} & 0 \\ 0 & a_{22} & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} u + \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} \\ y &= x_1\end{aligned}\tag{4}$$

in which,  $\Delta_1, \Delta_2, \Delta_3$  are bounded unknown uncertain parameters;  $R$  represents relative distance between the missile and target; and

$$\begin{aligned}
a_{11} &= -\frac{2\dot{R}}{R}, a_{12} = -\frac{57.3qSC_\gamma^\alpha + P}{mR}, a_{22} = -\frac{57.3qSC_\gamma^\alpha + P}{mV}, \\
a_{32} &= \frac{57.3qSLm_z^\alpha}{J_z}, a_{33} = \frac{qSL^2m_z^{\bar{\alpha}}}{VJ_z}, b_3 = \frac{57.3qSLm_z^{\delta}}{J_z}.
\end{aligned} \tag{5}$$

The aim of this paper is to design an appropriate controller for the uncertain control system (4), which makes the closed loop control system (4) globally uniformly asymptotic stable and the output of the closed loop control system (4) asymptotically approach zero.

### 3. Controller Design

In this section, we design the control law for system (4) based on Lyapunov stability theory. The control law can be presented in the following form:

$$u = -\frac{1}{b_3x_3}(a_{12}x_1x_2 + x_1\Delta_1 + x_2\Delta_2 + a_{32}x_2x_3 + a_{33}x_3^2 + x_3\Delta_3) \tag{6}$$

**Theory 1** Control law in (6) make the air-to-air missile uncertain control system (4) globally uniformly asymptotic stable.

**Proof** Choose a Lyapunov function for the system (4) as following:

$$V = \frac{1}{2} \sum_{i=1}^3 x_i^2 \tag{7}$$

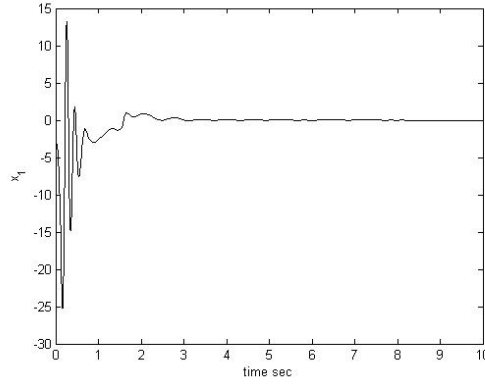
and its derivative along the systems (4) is as follows

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^3 x_i \dot{x}_i \\
&= x_1 \dot{x}_1 + \dot{x}_2 x_2 + \dot{x}_3 x_3 \\
&= a_{11}x_1^2 + a_{12}x_1x_2 + x_1\Delta_1 + a_{22}x_2^2 + x_2\Delta_2 \\
&\quad + a_{32}x_2x_3 + a_{33}x_3^2 + x_3\Delta_3 + b_3u \\
&= a_{11}x_1^2 + a_{22}x_2^2 \leq 0
\end{aligned} \tag{8}$$

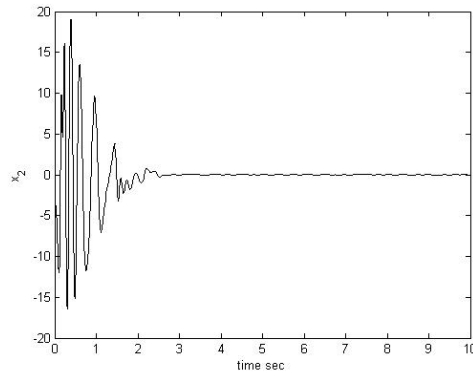
According Lyapunov stability theory, control system (4) is globally uniformly asymptotic stable.

### 4. Numerical Experiment

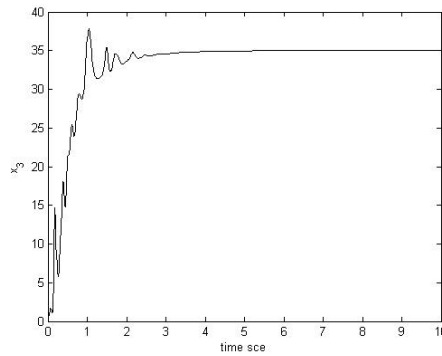
To verify the effectiveness of the integrated guidance and control law, numerical experiment is carried out for the proposed controller. The results are given in Figure1- Figure3.



**Figure 1:** dip angle rate curve



**Figure 2:** attack angle curve



**Figure 3:** pitch angle rate curve

Our aim is to make the closed loop of the air-to-air missile control system globally uniformly asymptotic stable and the output of the closed loop control system asymptotically approach zero. It can be seen from Figure1-Figure3 that the proposed controller has the advantages of short interception time, little target missing. The rudder deflection angle of the missile varies smoothly throughout the flying process, the variation of the attack angle and sideslip angle is also stable, and the amplitude of the fluctuation is also small. Especially in the near impact point, the missile's rudder angle and the angle of attack and sideslip angle without divergent trend.

Therefore, the proposed controller is efficient, real-time and robust for the air-to-air missile control system.

## **5. Conclusions**

In this paper, the guidance and control problem of the air-to-air missile system is studied. A nonlinear, coupling and uncertain dynamic model of the air-to-air missile with six degrees of freedom is considered. Then, based on Lyapunov stability theory, a Lyapunov function is employed, and a controller is designed for the air-to-air missile. To verify the effectiveness of the integrated guidance and control law, we carry out its simulation and their analysis show preliminarily that the integrated guidance and control law can guarantee the accuracy of the air-to-air missile hitting an arbitrarily maneuvering target and guarantee the stability of its attitude. Thus it is effective for interception.

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