

# Adaptive Vibration Control for Cable-Bridge Coupled Uncertain Nonlinear System

Yanjun Liang<sup>a\*</sup>, Junqi Liang<sup>b</sup>, Zhongsheng Wang<sup>c</sup>

<sup>a</sup>*School of Computer and Information Engineering, Anyang Normal University, Anyang 455000, China*

<sup>b</sup>*General Teaching Office, Shanghai Sunking Construction Management & Consulting Co. Ltd., Shanghai 200070, China*

<sup>c</sup>*Department of Automation, Guangdong Polytechnic Normal University, Guangzhou 510000, China*

<sup>a</sup>*Email: myluck0404@126.com*

<sup>b</sup>*Email: 105495707@qq.com*

<sup>c</sup>*Email: 84963337@qq.com*

## Abstract

Considering buffeting loading on the structure, technique of adaptive vibration control for long-span cable-bridge structure is developed. Model of vibration control for uncertain nonlinear systems of long-span cable-bridge structure is established; buffeting loading systems of long-span bridge structure is created by weighted amplitude wave superposition method. In order to depress buffeting loading influence of the wind-induced vibration for the structure and improve the robust performance of the vibration control, based on the semi-active vibration control devices and using adaptive control approach, an adaptive vibration controller and adaptive control laws for the uncertain parameters are designed. Numerical simulation results illustrate the effectiveness of the proposed technique.

**Keywords:** adaptive control; vibration control; uncertain system; nonlinear system; cable-bridge coupled.

## 1. Introduction

With the gradual increase volume of traffic, bridge plays an more and more important part in traffic system. Affected by wind, vehicles, pedestrians and seism loads, structural vibration of bridges is inevitable. Severe and complicated vibration of bridge structure, increase mechanical and fatigue damage, shortens the service life of the bridge, and reduces the security of traffic, so the bridge structure is widely studied by many scholars.

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\* Corresponding author.

The most study to bridge structure focuses on dynamic behavior [1-4], fatigue damage [4-6] and reliability analysis [7-8]. Using frequency and time domain methods in paper [1], dynamic characteristics of a laboratory bridge model are determined by operational modal analysis. According to the actual project, the effects of bridge width and bridge deck pavement thickness on dynamic characteristics of hinge joint voided slab bridge are researched in paper [2]. Considering wind, railway, and highway loadings in paper [3], dynamic stress analysis of long suspension bridges is presented. In paper [4], the dynamic properties of a decommissioned timber bridge are measured in the laboratory to observe the characteristics of the lowest few mode shapes as the support stiffness is varied to simulate deterioration of the pile supports, and an evaluation method is proposed to quantitatively assess the foundation competence of timber bridges. To avoid heavy interventions for strengthening of bridge deck slabs in paper [5], an improved building material is used, fatigue tests for the determination of the fatigue behavior of beams reinforced by this building material is presented, then a technique is developed to reduce the use costs as well as life cycle costs. Paper [6] presents the performance of an instrumented concrete bridge deck that developed longitudinal cracks that run along the entire length of the bridge, and the affect to their fatigue life is monitored. To analysis the reliability of Dongjiang Bridge in paper [7], a new analysis method that combined with each advantage of some common reliability computing method was put forward. The bridge is modeled as a single span simply supported Euler-Bernoulli beam and the vehicle is modeled as a single degree of freedom system in paper [8], and they present a reliability analysis of a simply supported bridge deck subjected to random moving and seismic loads. The previous studies are most about vibration response, monitor and analysis [9-12], vibration control for bridge structure has not been seen.

In this paper, the problem of optimal vibration control for long-span bridge structure affected by buffeting loading is studied. Employing weighted amplitude wave superposition method, buffeting loading forces on long-span bridge structure is created. Model of vibration control systems with uncertain parameters for long-span bridge structure is established. Adaptive vibration controller for the uncertain nonlinear system is designed. The numerical simulation demonstrated the control effect of the proposed controller.

## **2. Buffeting Loading**

The stiffness of bridge structure reduces, as the span of the bridge becomes longer, and the characteristics of flexible structure appear obviously. The effects of wind loading on bridge structure can not be neglected in engineering, and many scholars turn their attention to the effects of wind-induced vibration on structure. Bridge buffeting is a sort of random vibration of the bridge structure affected by fluctuating wind. Fluctuating wind field induced by natural wind can be described by ergodic and stationary Gauss random process, and can be regarded as a single variable four dimensions random field in mathematics.

We employ weighted amplitude wave superposition method to describe buffeting loading forces on long-span bridge structure. According weighted amplitude wave superposition method, buffeting loading forces on long-span bridge structure can be simulated by superposition of weighted amplitude waves. The buffeting force of the  $j$ th composition wave on long-span bridge structure is as following:

$$\begin{aligned} \bar{p}_j &= A_j \sin(\omega_j t + \varphi_j), \\ j &= 1, 2, \dots, r \end{aligned} \quad (1)$$

in which  $A_j$  and  $\omega_j$  is the amplitude and frequency of the  $j$ th wave component,  $\varphi_j$  is the random phase angles uniformly distributed in  $0 \leq \varphi_j < 2\pi$ . Let  $\tilde{\mathbf{p}}(t) = [\bar{p}_1, \bar{p}_2, \dots, \bar{p}_r]^T$ , we have:

$$\begin{aligned} \ddot{\bar{p}}_j &= -\omega_j^2 \bar{p}_j, \quad j = 1, 2, \dots, r, \\ \ddot{\tilde{\mathbf{p}}}(t) &= -\text{diag}\{\omega_1^2, \omega_2^2, \dots, \omega_r^2\} \tilde{\mathbf{p}}(t) \\ &= -\bar{\boldsymbol{\Omega}}^2 \tilde{\mathbf{p}}(t) \end{aligned} \quad (2)$$

in which  $\bar{\boldsymbol{\Omega}} = \text{diag}\{\omega_1, \omega_2, \dots, \omega_r\}$ . Let  $\mathbf{w}(t) = [\bar{\mathbf{p}}(t) \dot{\tilde{\mathbf{p}}}(t)]^T$ , then

$$\begin{aligned} \dot{\mathbf{w}}(t) &= \begin{bmatrix} \mathbf{0} & \mathbf{I}_r \\ -\bar{\boldsymbol{\Omega}}^2 & \mathbf{0} \end{bmatrix} \mathbf{w}(t) = \bar{\mathbf{G}}\mathbf{w}(t), \\ \tilde{\mathbf{p}}(t) &= [\mathbf{I}_r \quad \mathbf{0}]\mathbf{w}(t), \\ \mathbf{p}(t) &= [1, \dots, 1]\tilde{\mathbf{p}}(t) \\ &= \underbrace{[1, \dots, 1, 0, \dots, 0]}_r \mathbf{w}(t) = \bar{\mathbf{F}}\mathbf{w}(t) \end{aligned} \quad (3)$$

where  $\bar{\mathbf{F}} = \underbrace{[1, \dots, 1, 0, \dots, 0]}_r$ ,  $\bar{\mathbf{G}} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_r \\ -\bar{\boldsymbol{\Omega}}^2 & \mathbf{0} \end{bmatrix}$ ,  $\mathbf{I}_r$  is the  $r$ -order unit matrix,  $\mathbf{0} \in R^{r \times r}$  is the zero matrix.

So the total buffeting loading force acting on the bridge structure can be generated by the system

$$\begin{aligned} \dot{\mathbf{w}}(t) &= \bar{\mathbf{G}}\mathbf{w}(t), \\ \mathbf{p}(t) &= \bar{\mathbf{F}}\mathbf{w}(t) \end{aligned} \quad (4)$$

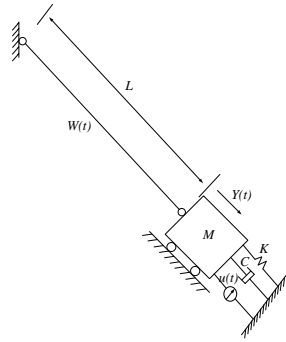
### 3. Mechanical Model and Dynamical System

Considering cable-bridge structure with semi-active tune mass damper devices, the mass matrix of the first modal, the damping matrix of principal mode and the stiffness matrix are  $M$ ,  $C$  and  $K$ .

Considering cable-bridge structure with semi-active tune mass damper devices, the deck is simplified as lumped mass affected by cable end, stiffness and damping of the deck is  $M$ ,  $C$  and  $K$ . Cable-bridge coupled vibration induced by buffeting loading is decomposed into axial direction motion and perpendicular to cable axis motion. The problem of axial direction motion is studied in this paper. The mechanical model of cable and deck is shown in Figure 1.

In order to simplified the problem and reflect the essence of the vibration control for the cable-bridge structure, we make some fundamental assumptions as follows.

- 1) Flexural stiffness, torsional stiffness and shear stiffness is disregarded;
- 2) The gravity sag curve is considered as parabola;
- 3) Constitutive relation of deformation for the cable satisfies Hooke's law and is uniform for each point.
- 4) The effect of tower vibration on cable is disregarded.



**Figure 1:** mechanical model of cable and deck

Generally, the fundamental mode is in dominant position, therefore we consider the first mode as the main object to study the cable-bridge coupled vibration control problem. The dynamical system of cable-bridge coupled vibration induced by buffeting loading is as following:

$$\begin{aligned} \ddot{W}(t) + (\omega_1^2 + a_3 Y(t))W + a_1 W^3(t) + a_2 W^2(t) + a_4 Y(t) &= 0, \\ u(t) + p(t) = \ddot{Y}(t) + 2\omega_2 \xi \dot{Y}(t) + \omega_2^2 Y(t) + a_5 W(t) + a_6 W^2(t) & \end{aligned} \quad (5)$$

in which,  $W$  is cable displacement from the equilibrium position;  $Y$  is the displacement of the cable end, namely, the deck displacement along the axial direction of the cable;  $\omega_1$  and  $\omega_2$  are the intrinsic frequencies of the cable and deck, respectively;  $\xi$  is the damping ratio of the deck;  $a_i (i = 1, 2, \dots, 6)$  are uncertain parameters, and they are determined by the factors such as the mass per unit length and the dip angle of the cable, and the regular function of the deck vibration mode.

Choose state variables for the cable-bridge coupled vibration control system (5) in the following:

$$x_1(t) = W(t), x_2(t) = \dot{W}(t), x_3(t) = Y(t), x_4(t) = \dot{Y}(t), \quad (6)$$

and the system (5) is rewritten in the state-space representation: and the system (5) is rewritten in the state-space representation:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{f}(\mathbf{x}) + \mathbf{D}p(t), \\ \mathbf{x}(0) &= \mathbf{x}_0, \end{aligned} \quad (7)$$

in which

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_1^2 & 0 & -a_4 & 0 \\ 0 & 0 & 0 & 1 \\ -a_5 & 0 & -\omega_2^2 & -2\omega_2\xi \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad (8)$$

$$f(x) = \begin{bmatrix} 0 \\ -a_1x_1^3(t) - a_2x_1^2(t) - a_3x_3(t)x_1(t) \\ 0 \\ -a_6x_1^2(t) \end{bmatrix}.$$

#### 4. Design of Adaptive Control

In order to compensate the influence of uncertain parameters  $a_i$  ( $i=1,2,\dots,6$ ) in the nonlinear system (7), we construct adaptive parameters  $\hat{a}_i(t)$  :

$$\begin{aligned} \bar{a}_i(t) &= a_i - \hat{a}_i(t), \\ i &= 1, 2, \dots, 6, \end{aligned} \quad (9)$$

in which,  $\hat{a}_i(t)$  ( $i=1,2,\dots,6$ ) are estimations for  $a_i$  ( $i=1,2,\dots,6$ ), and  $\bar{a}_i$  ( $i=1,2,\dots,6$ ) are errors.

We design adaptive vibration controller and adaptive control laws for nonlinear system (7) as following:

$$\begin{aligned} u(t) &= \frac{1}{x_4(t)} [[\omega_1^2 - 1 + \hat{a}_2x_1(t) + \hat{a}_1x_1^2(t) + \hat{a}_3x_3(t)]x_1(t)x_2(t) \\ &\quad + [\hat{a}_4x_2(t) - x_4(t)]x_3(t) + [\hat{a}_5 + \hat{a}_6x_1(t)]x_1(t)x_4(t) \\ &\quad + \omega_2^2x_3(t) - p(t) \\ \dot{\hat{a}}_1(t) &= -x_1^3(t)x_2(t), \\ \dot{\hat{a}}_2(t) &= -x_1^2(t)x_2(t), \\ \dot{\hat{a}}_3(t) &= -x_1(t)x_2(t)x_3(t), \\ \dot{\hat{a}}_4(t) &= -x_2(t)x_3(t), \\ \dot{\hat{a}}_5(t) &= -x_1(t)x_4(t), \\ \dot{\hat{a}}_6(t) &= -x_1^2(t)x_4(t). \end{aligned} \quad (10)$$

**Theory 1** Controller and adaptive control law in (10) make the cable-bridge coupled vibration control system (7) globally uniformly asymptotic stable.

**Proof** Take a Lyapunov function as following

$$V = \frac{1}{2} \left( \sum_{i=1}^4 x_i^2 + \sum_{i=1}^6 \bar{a}_i^2 \right) \quad (11)$$

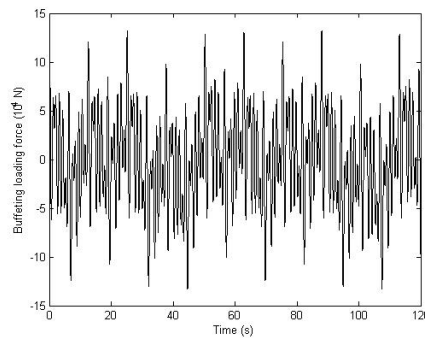
for the pendulum control systems (10), and its derivative along the systems (10) is as follows

$$\begin{aligned}
 \dot{V} &= \sum_{i=1}^4 x_i \dot{x}_i + \sum_{i=1}^6 \bar{a}_i \dot{\bar{a}}_i \\
 &= x_1 x_2 + x_2 [-(\omega_1^2 + a_3 x_3) x_1 - a_1 x_1^3 - a_2 x_1^2 - a_4 x_3] \\
 &\quad + x_3 x_4 + x_4 [-2\omega_2 \xi x_4 - \omega_2^2 x_3 - a_5 x_1 - a_6 x_1^2 + u + p] - \sum_{i=1}^6 \bar{a}_i \dot{\bar{a}}_i \\
 &= [-a_3 x_1 x_3 - a_1 x_1^3 - a_2 x_1^2 - a_4 x_3] x_2 + [-a_5 x_1 - a_6 x_1^2] x_4 - \sum_{i=1}^6 \bar{a}_i \dot{\bar{a}}_i \\
 &\quad + [\hat{a}_3 x_1 x_3 + \hat{a}_1 x_1^3 + \hat{a}_2 x_1^2 + \hat{a}_4 x_3] x_2 + [\hat{a}_5 x_1 + \hat{a}_6 x_1^2] x_4 \\
 &= [-\bar{a}_3 x_1 x_3 - \bar{a}_1 x_1^3 - \bar{a}_2 x_1^2 - \bar{a}_4 x_3] x_2 + [-\bar{a}_5 x_1 - \bar{a}_6 x_1^2] x_4 - \sum_{i=1}^6 \bar{a}_i \dot{\bar{a}}_i \\
 &= -2\omega_2 \xi x_4^2
 \end{aligned} \tag{12}$$

According Lyapunov stability theory, control system (7) is globally uniformly asymptotic stable.

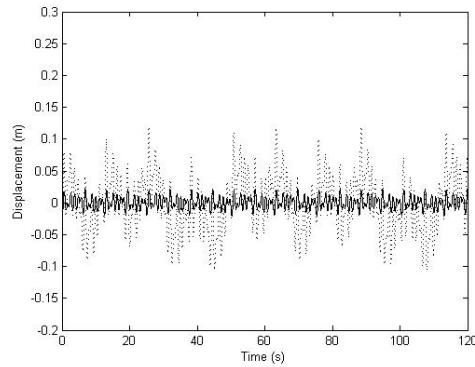
### 5. Numerical Experiment

We using a road suspension bridge<sup>[10]</sup> with three towers located in Yangtze River, whose span arrangement is 360m+1080m+1080m+360m=2880m, whose stiffening beam 3.5m height and 38.5m width with a tuyere consisted of a top and a bottom inclined plates is closed flat steel box beam, whose cable bent towers are 178.3m high, whose two main cables crosswise space between is 35.8m, whose main part of tower is filled by concrete and is H model, whose middle tower is supported by expanded triangle. The frequency of the first modal is 0.470Hz, and damping ratio is 0.02. Employing Matlab software, numerical experiment is carried out for the proposed optimal vibration controller. Figure 2 shows the buffeting loading force curve.

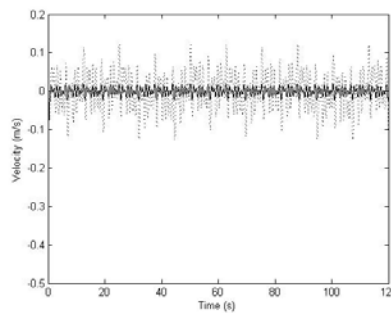


**Figure 2:** buffeting loading force curve

The main purpose of vibration control of long-span cable-bridge is to reduce the deck displacement which indicates the limit of the deck motion and to reduce the deck velocity to ensure the road holding ability for vehicles and pedestrians. So, to evaluate effectiveness of the proposed control strategy, the deck displacement and velocity are considered. Then, the corresponding curves of open loop system and the system controlled by the proposed adaptive vibration controller are compared, and shown in Figs. 3-4.



**Figure 3:** displacement curves



**Figure 4:** velocity curves

The curves of displacement are shown in Figure 3, velocities are in Figure 4, in which, dotted lines represent the open loop results of the deck of long-span bridge systems, and solid lines describe the results of the long-span bridge systems controlled by the proposed control strategy. It can be seen from these numerical results that the proposed adaptive controller is efficient, real-time and robust in reducing displacement and velocity of the deck, thereby ensures safety of the long-span bridge and enhances passing vehicle ride comfort.

## 6. Conclusions

The influence of wind-induced vibration on the long-span bridge structure cannot be neglected in construction phase or in operation stage. Based on the semi-active vibration control devices, the impact of buffeting loading is considered in this paper, and adaptive vibration controller and adaptive regulators are designed for the long-span bridge. Numerical simulation results demonstrated that the proposed strategy is efficient, real-time and robust in reducing the vibration induced by buffeting loading.

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