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A Comprehensive Study of Optimal Linear Pre-coding Schemes for a Massive Mu-MIMO Downlink System; a Survey

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Abstract

Massive Multi-User Multiple input Multiple Output (MU-MIMO) has become one of the leading area in terms of research in wireless communication due to the fact that the number of users and applications have increased tremendously, among all the aspects of massive mu-mimo systems out there, this manuscript focuses on linear precoding for downlink (DL) system at the base station(BS). This manuscript provides a comprehensive survey of precoding techniques for downlink transmission under a single-cell (SC) scenario. In a single-cell (SC) scenario the performance of the precoding techniques, Zero-Forcing (ZF), Match Filter (MF),Truncated polynomial Expansion and Regularized Zero-Forcing (RZF) are analyzed and compared in terms of Spectral Efficiency, and Achievable sum rate, a Rayleigh fading channel under perfect channel state information (CSI) is assumed. The template is used to format your paper and style the text. All margins, column widths, line spaces, and text fonts are prescribed; please do not alter them.

Keywords: Spectral efficiency; downlink; truncated polynomial expansion; precoding; zero forcing; match filter; Rayleigh fading; channel state information.

1. Introduction

In 1897, Guglielmo Marconi first demonstrated radio’s ability to provide continuous contact with ships sailing the English Channel since then wireless communication has gone through series of generations .With rapid development of information and communication technologies (ICT) particularly the wireless communication technology, it is becoming very necessary to analyze the performance of different generations of wireless technologies.

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To achieve this, technology has advanced in each generation of communication architecture, aiming to improve network capacity, efficiency, and reliability [3]. As one of the foremost promising wireless communication solutions, multiple-input multiple-output (MIMO) technology has been broadly utilized in IEEE 802.11ac/n (WiFi), IEEE 802.16e (WiMAX), LTE/LTE-A (4G), and other protocols and systems. The development of MIMO wireless communication originated from the idea of point-to-point MIMO communication also known as single-user beam-forming this is when the point to point communication device have multiple antennas, there is another MIMO known as MU-MIMO (Multiuser MIMO) this MIMO has a big advantage over the point to point MIMO , this is where a basestation has multiple antennas serving single-antenna end users MU-MIMO communication is more advantageous than point to point MIMO communication in terms of enabling cheap single-antenna terminals and being less sensitive to propagation environment. However, a conventional MU-MIMO link suffers from interuser interference, which considerably limits the performance of the system. Although techniques such as dirty paper coding (DPC) can reduce the effect of interuser interference, system implementation complexity could be sacrificed as a tradeoff [3]. Massive MIMO, unlike conventional MIMO, employs large number of antennas at the base station (BS) side on the order of more than 100 antennas that simultaneously serve a few single antenna users, e.g., tens of users, using the same time-frequency resources. That is also known in the literature as multiuser multiple input multiple output abbreviated by MU-MIMO. This configuration of very large MU-MIMO system implies that the single-antenna terminals do not cooperate neither for transmission nor for reception. Furthermore, massive MIMO technology provides both high data rates and spectrum efficiency [1]. It additionally yields a good higher power efficiency than in typical MIMO with less inter-user interference also. All of those options aboard millimeter wave deployment guarantee reliable and enabling technology for the subsequent generation of wireless communication systems. A very large MIMO (multiple-input multiple-output) improves the spectral efficiency (SE) of cellular networks by spatial multiplexing of a large range of user equipment (UEs) per cell. It is thus considered an essential time-division duplex (TDD) technology for the next generation of mobile networks. The main difference between Massive MIMO and classical multiuser MIMO is the large number of antennas, M, at each base station (BS). if the BS uses many antennas, the interuser interference can be largely eliminated because of the asymptotic orthogonality among the MIMO channels, which will yield huge performance improvement other benefits of massive MIMO include improving spectral efficiency (SE) without BS densification, increasing energy efficiency (EE). CSI is estimated by linear channel estimators through uplink pilot transmissions, that leverages the channel reciprocity. In TDD operation, the time needed to amass CSI depends on the quantity of users however independent of the quantity of BS antennas. Thus, TDD is a more preferable operating mode for massive MIMO systems than the frequency-division duplexing (FDD) mode, where the amount of time required by FDD to obtain CSI relies on the excessive number of BS antennas [3]. However, the uncontrollable wireless propagation environment usually makes it difficult to obtain reliable CSI [2]. Although perfect CSI is usually unavailable at the transmitter, the performance of downlink transmission largely depends on CSI and the corresponding pre-coding technique employed. Lately, several research papers related to MIMO system precoding has been published this includes papers like [5, 7] whose scheme was based on maximizing the signal to leakage Ratio in the downlink at the (BS) basestation, other papers such as [10,11] also uses a (ZF) zero forcing approach to cancel the CCI completely this approach turns to be superior but there is a strong restriction in system configuration, precoding was also used in[6] to reduce the ICI in a MIMO-OFDM, so precoding happens to be in a number of MIMO systems. The rest of the paper is organized as follows. The next section describes the Massive MU-MIMO system considered in the paper, where the system model is derived and the optimal precoding techniques are studied. Section 3 gives a comprehensive study of the system performance analyzes where the corresponding achievable sum rate and spectral efficiency are discussed. Evaluation and simulation results are demonstrated in section 4. Section 5 entails the conclusion and future research plan.

2. System Model

Let us consider a single-cell (SC) massive multiuser multiple input multiple output (MU-MIMO) downlink (DL) system where the base station (BS) is equipped with massive number of antennas M transmitting to K single antenna mobile users. The channel is a Rayleigh Fading MIMO channel with the assumption of a perfect channel state information (CSI) [17]. In practical scenario the achievable gain depends on the precoding measures which are the focus of this manuscript, let $\mathbf{x}$ be the transmitted vector for the K user during the downlink transmission where M > K, let $\mathbf{A} \in \mathbb{C}^{\mathbf{M}\times\mathbf{N}}$ be the linear precoding matrix, $\mathbf{s} \in \mathbb{C}^{\mathbf{K}\times\mathbf{1}}$ is the transmitted information before precoding and $\mathbf{P_l}$ is the average transmit power at the base station (BS) , now let $\mathbf{H}$ denote the channel matrix, whereby the element of $\mathbf{H}$ are independent and identically distributed (iid) complex Gaussian variables with zero mean and unit variance. From the above variables the transmitted signal vector can be deduced as

$$x = \sqrt{P_l}As \quad (1)$$

Where n is the Gaussian white noise and interference. To appease the power coercion at the BS, the transmitted signal power is normalized as E|s|2 = 1 and the tr(AA^H) = 1. In TTD mode the downlink channel is simple the transpose of the channel matrix H [3] thus the set of signals received at the K are given by:

$$y = H^T x + n \quad (2)$$

$$y = \sqrt{P_l}AH^T s + n \quad (3)$$

2.1. Linear precoding scheme

In this part we will consider the following pre-coding schemes, Zero forcing (ZF), Match Filter (MF) , Regularized Zero Forcing (RZF) and Truncated Polynomial Expansion (TPE)

2.2. Zero forcing precoding (ZF)

Zero-forcing pre-coding (ZF) is a spatial signal processing in multiple antenna wireless devices. For downlink (DL), the ZF algorithm allows a transmitter to send data to desired users together with nulling out the directions to undesired users and for uplink (UL), ZF receives from the desired users together with nulling out the directions from the interference users [15]. It is an optimal pre-coding scheme in the absence of additive noise that is in the presence of additive noise this technique might amplify it [20] The ZF pre-coder according to paper

[3,16,17] is given by; for notation simplicity we will neglect the subscript k.

$$A = \sqrt{\sigma}H^{*}(H^{T}H^{*})^{-1} \tag{4}$$

Where σ is a scaling factor to normalize signal power.

$$A = H^{*}(H^{T}H^{*})^{-1} \tag{5}$$

Now we substitute equation (5) into that of (3) the receive signal then becomes.

$$y = \sqrt{P_{l}}H^{*}H^{T}(H^{T}H^{*})^{-1} \tag{6}$$

2.3. Match filter (MF)

MF pre-coder is also known as maximum ratio transmitter (MRT), which maximizes signal gain at the intended user or it is simply the conjugate transpose of the downlink channel matrix. MRT works well in the MU-MIMO system where the base station radiates low signal power to the users [18] Good energy efficiency (EE) can be achieved when the value of the N and K are close as shown in paper [19] however in the case of Massive MIMO the asymptotic property does not hold so such a system will suffer from inter-user interference. From [18,2] MRT pre-coder can be written as.

$$A = \sqrt{\sigma}H^{*} \tag{7}$$

Where σ is a scaling factor to normalize signal power.

$$A = H^{*} \tag{8}$$

Now we substitute equation (8) into that of (3) the receive signal then becomes.

$$y = \sqrt{P_{l}}H^{*}H^{T}s + n \tag{9}$$

2.4. Regularized zero forcing (RZF)

This pre-coding technique is popular due to its alternative number of names such as signal-to-interference-noise ratio (SINR), transmit Wiener filter, and signal-to-leakage-and-noise ratio (SLNR) Lagrangian optimization method is used for obtaining this precoder. Regularized Zero Forcing has been considered to be one of the best pre-coding techniques out there since it’s capable of trading off the advantages of both Zero forcing (ZF) and Maximum Ratio Transmitter (MRT). According to papers [2,18,22] RZF pre-coder is given by.

$$A = \sqrt{\sigma}H^{*}(H^{T}H^{*} + Q + \beta I)^{-1} \tag{10}$$

$$A = H^{*}(H^{T}H^{*} + Q + \beta I)^{-1} \tag{11}$$

Where σ a scaling factor to normalize signal power, $Q \in CN^{N\times N}$ is a Hermittian nonnegative matrix, β is a regularization factor. When $Q = 0$ and β approaches zero ($\beta \to 0$) then equation (11) becomes a ZF on the other hand When $Q = 0$ and β approaches infinity ($\beta \to \infty$) equation (11) becomes an MF, thus Q and β are arbitrary.Now we substitute equation (11) into that of (3) the receive signal then becomes

$$y = \sqrt{P_l} H^T H^* (H^T H^* + Q + \beta I)^{-1} s + n \tag{12}$$

2.5. Truncated polynomial expansion (TPE)

It is a recently proposed pre-coding technique; its main objective is to reduce the computational complexity of the regularized Zero Forcing (RZF) [3]. The concept of TPE originated from the Cayly-Hamilton theorem which states that the inverse of a matrix B of dimension M can be written as a weighted sum of its first M powers [23], so according to [23] the matrix B can be written as.

$$B^{-1} = \frac{(-1)^{M-1}}{\det(B)} \sum_{\ell=0}^{M-1} \alpha_\ell A^\ell \tag{13}$$

From (13) the coefficient of the characteristic polynomial is α_ℓ. Now by taking only truncated sum of the matrix powers a simplified precoding could be obtained known as TPE precoding. Interested researchers on truncated polynomial expansion (TPE) should refer to [23] for more details.

According to [3,23,24] Truncated Polynomial Expansion TPE precoding is given by ;

$$A = \sum_{\ell=0}^{J-1} \omega_\ell (H^T H^*)^j H^T \tag{14}$$

Where ω_ℓ is a scaler coefficient of polynomial precoder of order J. That is a good selection of ℓ ensures a proper passage between the low MF and the High RZF. Now we substitute equation (14) into that of (3) the receive signal then becomes.

$$y = \sqrt{P_l} H^T H^T \sum_{\ell=0}^{J-1} \omega_\ell (H^T H^*)^j + s + n \tag{15}$$

3. Performance Analysis

3.1. Achievable rate (RZF)

The achievable rate is one of the methods that can be used to determine system performance. Followed by Shannon theory in other words popularly known as Shannon capacity, this theorem talks about the maximum capacity rate which the transmitter can transmit over channel, the parameter of the channel are assumed to be Gaussian random process and the channel is assumed to be ergodic .The Shannon capacity theorem is given in paper [10] as.

$$C = log_2(1 + SNR) bits/s/Hz \tag{16}$$

DPC turns to give the highest capacity as compared to optimal linear pre-coding but it turns out to be difficult to implement. The DPC capacity technique is given in paper [5] as.

$$C_{DPC} = maxlog2det(\mathrm{I}_M + pHPH^H)\ bits/s/Hz \tag{17}$$

Now considering a Single-Cell (SC) massive multiuser multiple input multiple output (MIMO) system the transmitter must know the channel state information (CSI). Therefore CSI turns out to be the key factor in a Multiuser (MU) communication system. Thus the achievable sum rate in a Single-Cell (SC) downlink massive MU MIMO system with perfect CSI is given in [5] as

$$R_k = log_2(1 + SNR_K) bits/s/Hz \tag{18}$$

The achievable sum rate of the Zero forcing (ZF) pre-coder is given as [17]

$$R_{zf} = Klog_2\left[1 + P_l\left(\frac{M-K}{K}\right)\right] \tag{19}$$

The achievable sum rate of the Match filter (MF) pre-coder is given as [17]

$$R_{mf} = Klog_2\left[1 + \frac{P_l M}{K(P_l+1)}\right] \tag{20}$$

The achievable sum rate of RZF can be obtained by substituting the SNR of RZF in [25] into equation (18) interested readers can refer to [25][22] for more details. Now regarding the achievable sum rate of TPE pre-coding the SNR is given in [24] and the achievable sum rate can be found in [3].

3.2. Spectral efficiency

The SE of an encoding/decoding scheme is the average number of bits of information, per complex valued sample, that it can reliably transmit over the channel under consideration. the SE can be viewed as the average number of bit/s/Hz over the fading realizations.[26] In a single cell massive multiuser mimo system the spectral efficiency is given in [18] as.

$$\mathrm{S_e} = \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{A_k} \quad (\mathrm{bits/s/Hz}) \tag{21}$$

Where $\mathrm{S_e}$ is the spectral efficiency in bits/s/Hz and $\mathrm{A_k}$ is the achievable rate of user k,

4. Simulations and analysis

We considered a single cell massive MU-MIMO system, over Rayleigh fading channel with ZF, MF, TPE (J =

2, 4, 5) and RZF optimal linear pre-coding schemes. The simulation settings are elaborated below. Firstly we assigned an increasing number of antennas and constant number of users, after which we assigned an increasing number of user and constant number of antennas. However, the limitation of the scenarios are that the number of users K, is not greater than number of transmit antennas M that is M ≫K .All results are shown in terms of the achievable sum rate against the transmission power(SNR), the spectral efficiency against the number of antenna arrays, the spectral efficiency against the number of user.

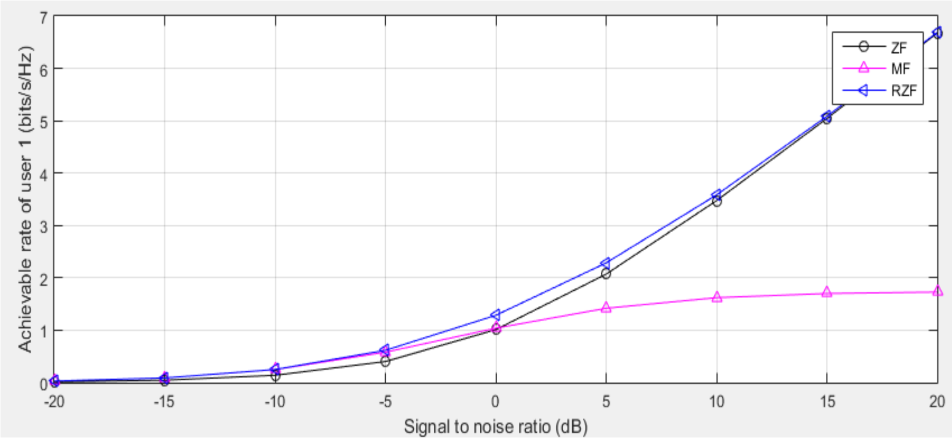

Figure 1: The achievable rate against SNR

From figure 1, it can be observed that at low SNR MF pre-coder performs better than ZF precoder, but at High SNR ZF pre-coder performs far better than MF, thus MF is preferable at low power an ZF at high power on the other hand it can be observed that RZF precoder has superior performce compared to both ZF and MF, this because the ZF precoder performs good at both high and low SNR, therefore RZF has an advantage of trading off both the advantages of ZF and MF precoder .

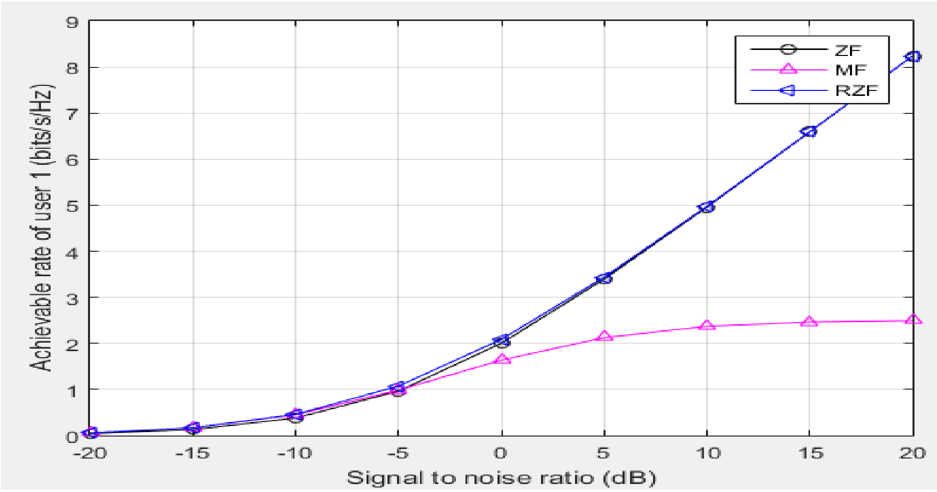

Figure 2: The achievable rate against SNR M= 50 K = 20

From figure2 we see that the performance of the system is better and it can be observed that increasing the number of antennas enabled the ZF to be used at Low SNR but looking at the MF nothing changed as to whether it can be used at High SNR but the RZF still has a superior performance with an increased antenna.

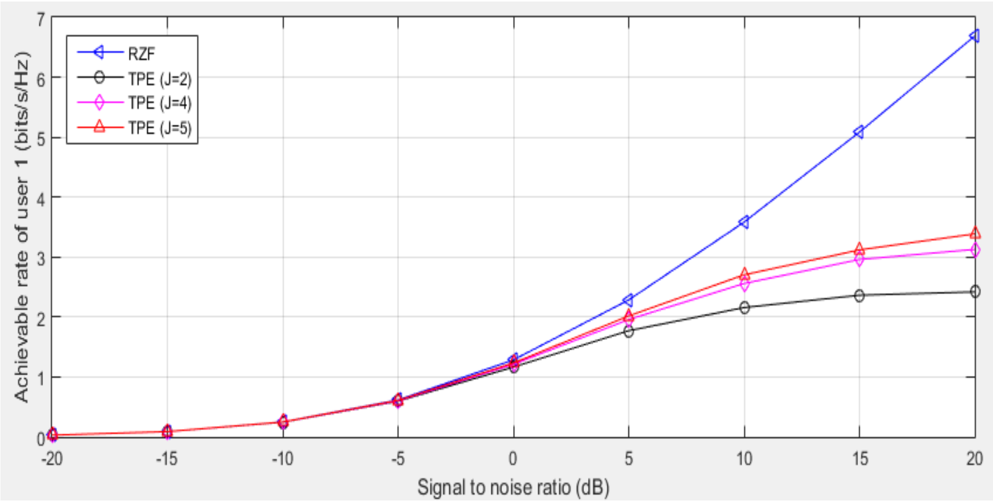

Figure 3: The achievable rate against SNR (TPE and RZF)

Looking at figure 3 it can be observed that the RZF performs better against the given values of the TPE (J=2,4,5) since it performs better at both low and high SNR but looking at it closely we can observe that The TPE increases with an increase in the J values so TPE not only can it perform better than RZF but it also reduces computational complexity of RZF.

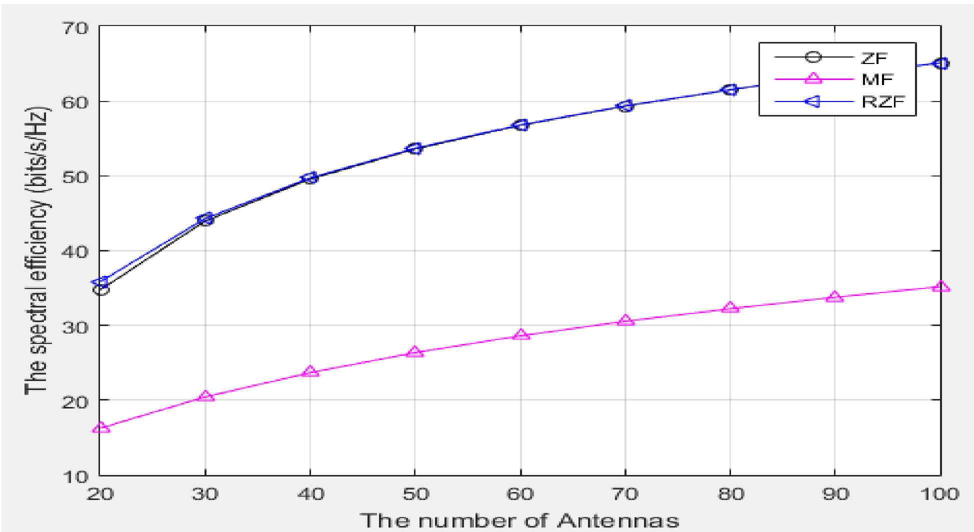

Figure 4: Spectral Efficiency against number of antennas

Figure4 and figure5 corresponds to figure 1, 2 and 3 that is as you increase the number of basestation antennas the SE of all system precoding increases. Figure4 is a plot between the SE and the number of basestation antennas so from figure4 we can observe that the SE increased to 20bits/s/Hz when the antennas were increased from 20 to 60, furthermore it can be observed that both the ZF and the RZF has a higher SE with increasing number of antennas as compared to the MF. Now from fig 5 it can be observed that even though the RZF appears to have a higher SE as compared to the TPE but it can also be observed that the TPE J values move towards the SE of the RZF with increase in the J values so the TPE not only does it perform good at both Low and High SNR it also has a higher SE.

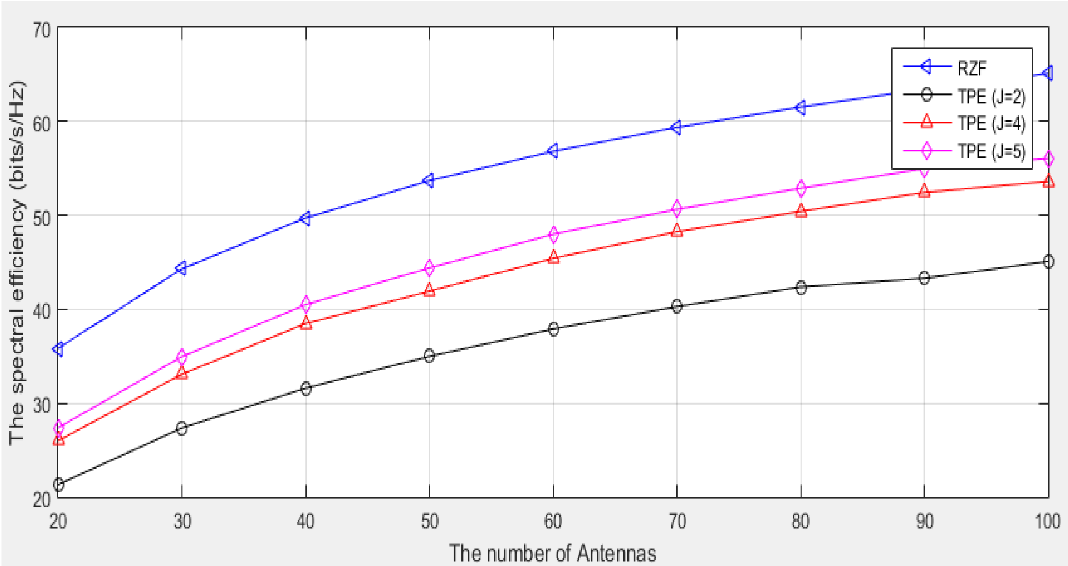

Figure 5: Spectral Efficiency against number of Antennas (TPE and RZF)

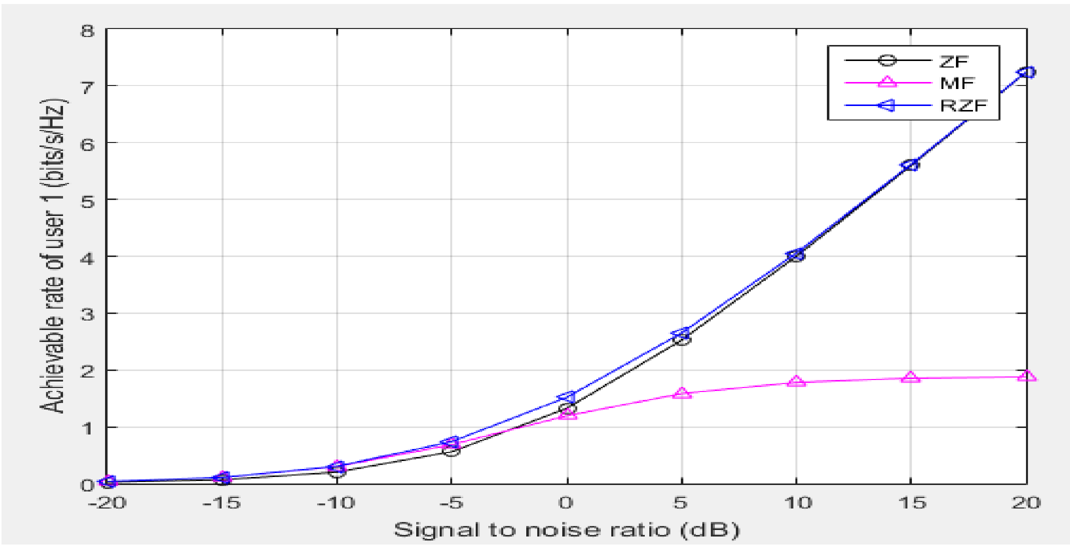

Figure 6: Achievable rate against SNR M= 50 K= 20

From figure 6 It can be observed that with increase in users M = 50, K = 20 the performance of the Massive MU MIMO system decreases this is due to the system suffering from inter-user interference , all the achievable rate of the linear precoding has been reduced but the MF still performs best at low SNR whiles ZF and RZF performs best at High SNR.

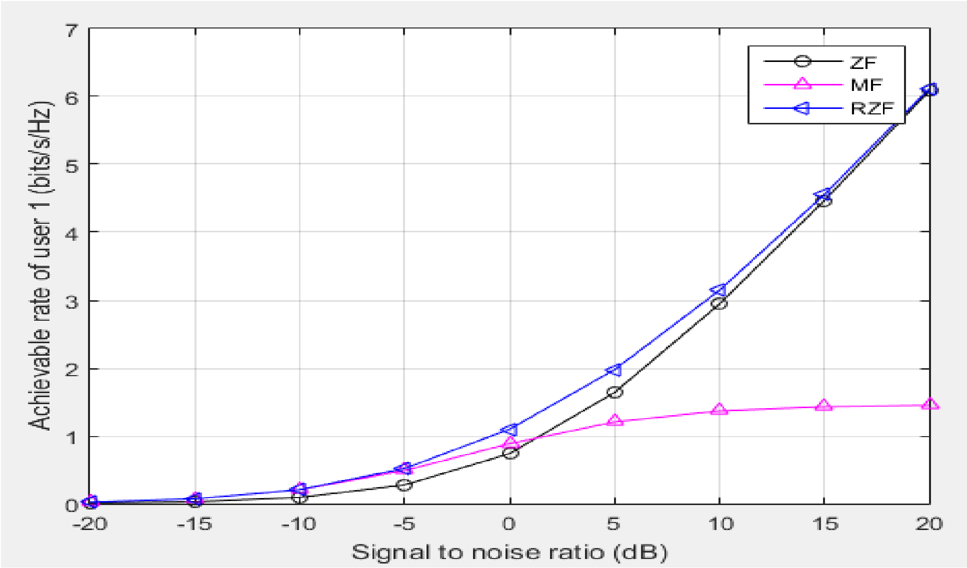

Figure 7: Achievable rate against SNR M=50 K= 30

Now looking at fig 7 we can see that the performance has decreased further with increasing number of users at a fix number of antennas that is, M = 50, K = 30 even though the entire linear precoding has been decreased further that is comparing fig6 to fig7 so the inter-user interference in fig7 is more than that of fig 6 this is due to the increase in users. Even though the achievable rate has been decreased further but still the Mf has a better performance at a Low SNR as compared to the others.

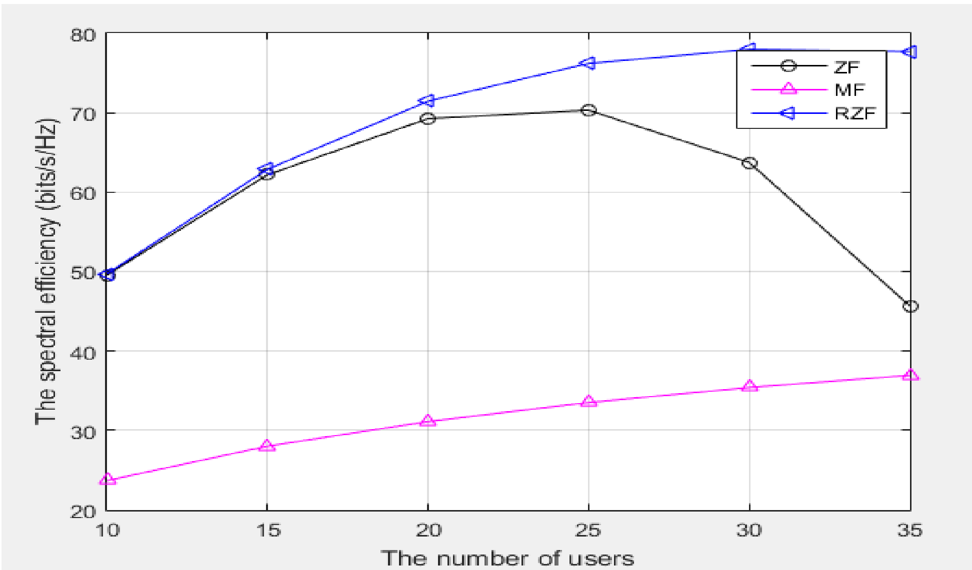

Figure 8: The Spectral Efficiency against the number of users

From figure8 it can be observed that both the ZF and the RZF has a higher SE with increase in users , the ZF kept on increasing till it hit 25 after which it decreased sharply, we can see a similar increase in the RZF but the RZF kept on increasing till 30 after which it attained a steady movement, now looking at the MF even though it has a low se at increasing users it kept on increasing at a steady pace.

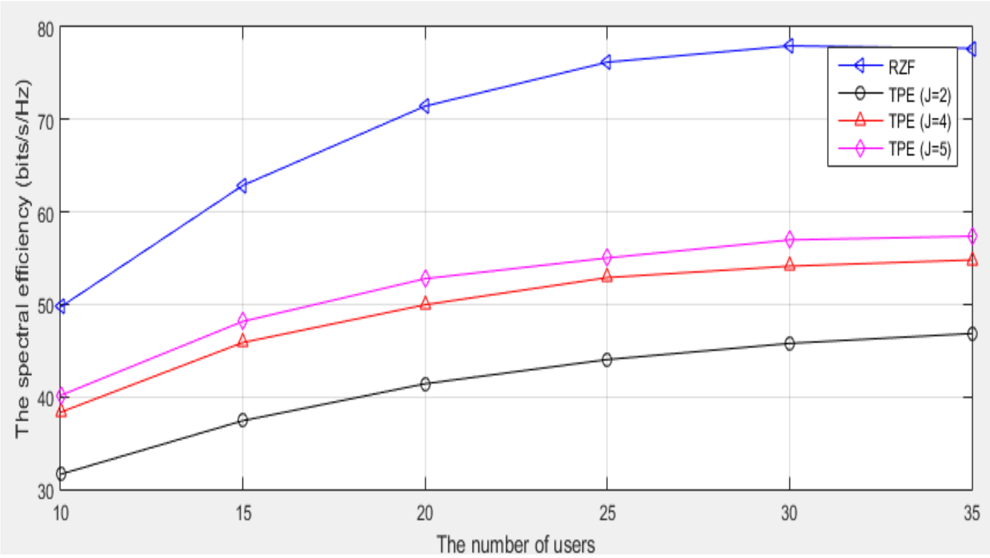

Figure 9: Spectral Efficiency against the number of users

Figure 8 shows that both the TPE and the RZF increases in SE with increase in users, it can be observed that the RZF has a higher SE as compared to the TPE but the TPE values(J =2,4,5) approaches the RZF this shows that choosing the right TPE value will enable the TPE to outperform the RZF and obtain a larger SE even with increasing users and again the RZF increased to 30 and then maintained a steady movement , on the other hand the TPE seems to be increasing slowly.

5. Conclusion

Massive MIMO is the key to the next wireless communication system, it shares light on the usefulness of increasing the spectral efficiency, this system is able to utilize the optimal linear precoding techniques that is the MF, ZF , RZF and TPE which is a newly introduced linear precoding technique at the basestation and using Channel estimation via uplink. Furthermore, this system offers advantages in terms of achievable rate and spectral efficiency. From the above simulation results we observed that MF performs better at low transmitting power and ZF performs better at high transmission power whiles the RZF performs better comparing it to the ZF and the MF , in contrast to the ZF the TPE turns to have the best performance since it's a suitable precoding scheme with a good overall performance and low computational complexity for a very large MU-MIMO downlink system. In the future we aim to do a research on the linear precoding schemes in a Multi-Cell scenario, we will do a comprehensive study and analyze the existing optimal linear precoding schemes that can be adapted in a very large MU MIMO system in relation to a Multi-Cell scenario (MC).

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