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Performance Guarantee of a Class of Continuous LPV System with Restricted-Model-Based Control

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Abstract

This paper considers the problem of the robust stabilisation of a class of continuous Linear Parameter Varying (LPV) systems under specifications. In order to guarantee the stabilisation of the plant with very large parameter uncertainties or variations, an output derivative estimation controller is considered. The design of such controller that guarantee desired $\mathcal{L}_2$ induced gain performance is examined. Furthermore, a simple procedure for achieving the $\mathcal{L}_2$ norm performance is proved for any all-poles single-input/single-output second order plant. The proof of stability is based on the polytopic representation of the closed loop under Lyapunov conditions and system transformations. Finally, the effectiveness of the proposed method is verified via a numerical example.

Keywords: Linear parameter-varying (LPV) system; Polytopic systems; Non-linear systems; Linear Matrix Inequalities; Performance control; $\mathcal{L}_2$ induced gain.

1. Introduction

In order to avoid processing the non-linear systems, there is the class of systems presented in parametric form where the stability study is carried out. The obtained results of this class are generally very applicable thanks to numerous existing systematic methods. Yet, since some information about the model is no longer used, these results suffer from an important conservatism. Among these systems, one notes the less specific class of non-linear systems like LPV systems [1, 2, 3] which allow to describe a global model as a set of linear sub-systems connected by functions. In addition, due to the variation and the non-linear nature of their parameters, the tuning of the complex plant still the subject of many researches. Even if the system model is available, its parameters identification is required for control law tuning. Unfortunately, the choice of the physical model structure, the identification of its parameters and then the experimental validation of the model are never simple and are time consuming.

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In order to control those plants and since internal states of most industrial systems cannot be directly measured and only their outputs are available for control purposes, output feedback controllers were proposed such as static output feedback [4, 5, 6], dynamic output feedback [7, 8, 9] and global sampled-data output feedback stabilization for a class of uncertain non-linear systems [10]. Often, the output feedback control involves two major problems. The first one is related to the feedback control law design for any plant to guarantee stability, tracking and performances. The second one is related to the non-availability of all variable parameters in real-time for implementation of complex control laws. In fact, the control law depends on some of the variable parameters and on the system output. If no information about these variable parameters is available, a constant output feedback control gain may be a solution. But, in general, this alternative can yield to conservative results. Therefore, PID controllers are often tuned with a simple (linear) nonphysical model [11, 12]. Due to their linear nature, the traditional PID controllers are very used for the stability synthesis. However, the performance guarantee is often limited to a state-space region around a given set-point which could lead, when a process has a large operating domain, to poor results.

In many practical control problems, since the state variable is not accessible for sensing devices and transducers are not available or are very expensive, the physical state variable of systems is partially or fully unavailable for measurement. In this cases, observer-based control should be considered to estimate the state. Thus, observer design has been the attracted field during the last decade and has turned out to be much more challenging than control problems. A great deal of research on the observer design for non-linear systems has been carried out such as the Kalman filter [13, 14, 15, 16], the Luenberger observer [17, 18, 19, 20], the H_∞ filter [21, 22, 23, 24, 25, 26, 27], the H_2 filter [28, 29], the mixed H_2/H_∞ filter [30, 31], the sliding mode observer [32, 33], the LQ control [34, 35, 36, 37] and the fuzzy observer-based control approaches [38, 39, 40, 41, 42, 43]. Each type of these observers has its own advantages and application fields.

Recently, some new results in the framework of *model free conytrol* have been introduced by Join and his colleagues [44]. The latter is signal based and don't require many information about the system. To implement this controller, the output derivatives have to be available in real time. In the literature, many variant of such controller have been developed (see [45, 46, 44, 47, 48, 49]). All these studies states that the stability as well as performances is ensured but none of them provides a standard proof on a class of system.

In this frame, the authors of [50] use the polytopic transformation and the model-free theory to resolve the problem of stability tuning method with restrict model of non-linear systems. It turns out that the obtained results require, for the controller tuning, less information on the system : only an estimation of the non-linear functions bounded, an estimation of the input gain and the dynamic order are required.

The originality in this work compared to the previous [50] is to show the existence of such controller for the stabilization of a class of linear parameter varying (LPV) systems under specifications. This controller has a fixed structure and only one parameter. The closed loop stabilization and performances are studied using the

Lyapunov theory, polytopic transformation, induced gain $\mathcal{L}_2$[a] and Linear Matrix Inequalities (LMI) conditions.

The main issues addressed in this work are:

• Providing stability conditions under specifications using the control law introduced by [50].

• Proving the existence of a stabilizing derivative/controller pair ensuring the desired specifications for all models belonging to a special class of second order LPV systems (all-poles single-input/single- output systems).

The rest of this paper is organized as follows. The Section II summarizes some of the main theoretical ideas which are shaping the model-free control presented on [50]. The third section is devoted to the stability synthesis under specifications of the closed loop. A particular class of second order non-linear systems is considered. In Section IV, some examples are illustrated in order to prove the usefulness of the proposed approach. The last section gives some conclusions and perspectives.

2. Restricted model-based control: general principle

This section will summarise some of the main theoretical ideas which are shaping the model-free control presented on [50]. We restrict ourselves for simplicity's sake to a class of systems with a single control variable u and a single output variable y i.e. Single-Input-Single-Output (SISO) systems with bounded time varying parameters. Since the model free control is sensitive to the presence of invariant zeros, the following class of SISO linear system with time varying parameters will be considered in the rest of this paper:

$$\begin{array}{l} y^{(n)}(t) = -a_0(t)y(t) - \cdots - a_{n-1}(t)y^{(n-1)}(t) + \alpha u(t), \\ |a_i(t)| < \overline{a}_i, \quad \forall i = 0..n-1 \end{array} \quad (1)$$

where $y(t) \in \mathbb{R}$ is the system output, $u(t) \in \mathbb{R}$ is the control input, α is the input gain and $a_i(t) \in \mathbb{R}$ are scalar time varying unknown parameters that their absolute values are bounded by $\overline{a}_i,\ \forall i = 0..n-1$.

This system can be represented by the following state space equation by choosing $x_m(t) = [y(t) \quad \dots \quad y^{(n-1)}(t)]^T$:

$$\begin{cases} \dot{x}_m(t) &= A_m(t)x_m(t) + B_m u(t) \\ y(t) &= C_m x_m(t) \end{cases} \quad (2)$$

where $A_m(t) \in \mathbb{R}^{n\times n}$ represents the dynamic, $B_m \in \mathbb{R}^{n\times n_u}$ is the system input matrix and $C_m \in \mathbb{R}^{n_y\times n}$ is the system output matrix with:

[a] The considered $\mathcal{L}_2$ norm is defined by: $||u(t)||_{\mathcal{L}_2} = \sqrt{\int_0^{+\infty} |u(t)|^2 dt}$

$$A_m(t) = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \dots & \dots & \dots & 1 \\ -a_0(t) & -a_1(t) & \dots & \dots & -a_{n-1}(t) \end{pmatrix} \quad (3)$$

$$B_m^T = (0 \quad 0 \quad \dots \quad 0 \quad \alpha), C_m = (1 \quad 0 \quad \dots \quad \dots \quad 0)$$

Since the time varying parameters of the state matrix of the system are bounded, it is more convenient to describe it by the polytopic form by considering the well known non-linear sector approach [51, 52]. This transformation allows to obtain a polytopic representation of the model in a compact and convex set of the state space:

$$\begin{cases} \dot{x}_m(t) & = & \sum_{i=1}^{N} \mu_i(t) A_{m_i} x_m(t) + B_m u(t) \\ y(t) & = & C_m x_m(t) \end{cases} \quad (4)$$

where the integer N is the number of subsystems and $A_{m_i} \in \mathbb{R}^{n\times n}$ are known matrices. The functions μ_i are the weighting functions depend on the time. These functions verify the convex sum property in the polytopic model domain of validity.

Since the idea of the controller structure is to nullify the system dynamic and then to replace it with the ideal dynamic for the closed loop, we consider the controller introduced by [50]:

Proposition 1. ***Control Law***

$$\begin{aligned} u(t) &= \hat{u}_n(t) + \hat{u}_r(t) \\ &= -\frac{1}{\hat{\alpha}} \hat{F}(t) + \frac{1}{\hat{\alpha}} (-K\hat{Y}(t) + k_0 r(t)) \end{aligned} \quad (5)$$

where

• $\mathbf{\hat{F}(t)}$ is an estimation of the "structure" function $F(t) = f(y,\ \dot{y}, \dots)$ containing all-poles dynamic with eventually some disturbances. Note that the function $F(t)$ MUST be control independent (no zero dynamics);

• $\hat{\alpha}$ is an approximation of the input gain α. Note that the choice of the parameter $\hat{\alpha}$ will be informed by the analysis of non-grouped terms in the structural function F;

• $\mathbf{\hat{Y}(t)} = [z_0(t) \quad \dots \quad z_{n-1}(t)]^T$ is a vector composed of $z_i(t)$: the estimations of the successive derivatives of the system output $y^{(i)}(t)$.

• $n \in \mathcal{N}^*$ is the order of the differential equation and it is supposed known.

• $\mathbf{r(t)}$ is the reference;

• $\mathbf{K} = [k_0,\ \dots,\ k_{n-1}]$ is a vector composed of the coefficients k_i of the desired dynamic of the closed system given by specifications such that:

$$y_r^{(n)}(t) = -k_0 y_r(t) - \cdots - k_{n-1} y_r^{(n-1)}(t) + k_0 r(t). \qquad (6)$$

Remark 1. The latter controller is based on the model-free control introduced by Join and his colleagues [44] were authors proposed an intelligent controller named : Intelligent Proportional-Integral-Derivative (iPID). The latter is signal based and don't require many information about the system. The model free control is based on a local modelling, constantly updated, from the only knowledge of the input-output system behaviour. To implement this controller, the output derivatives has to be available or estimated in real time.

The presented controller gives a perfect closed loop only if a good estimation of the output derivatives and of the function F(t) are available. In order to set a simple solution, for design purpose, a simple filtered derivative approach has been considered in [50] as follows:

$$\begin{cases} \frac{z_0(s)}{y(s)} & = & \frac{1}{\tau s+1} \\ \frac{z_i(s)}{y(s)} & = & \left(\frac{s}{\tau s+1}\right)^i \quad \forall i = 1..n-1 \end{cases} \qquad (7)$$

This estimator is causal and ensures a good estimation if its parameter τ is sufficiently smaller than the fastest dynamic of the system. It provides the successive estimations $z_i(t)$ of $y^{(i)}(t)$ for all $i \in \{1, \ldots, n-1\}$ where n presents the system order.

Finally, as a result the controller/derivative pair composed of the control law (5) and the estimator (7) is defined by [50] as follow:

Proposition 2. *The state representation of the controller and its derivative can be given by:*

$$\begin{cases} \dot{x}_e(t) & = & A_o x_e(t) + B_{o_1} y(t) + B_{o_2} r(t) \\ u(t) & = & C_o x_e(t) + D_{o_1} y(t) + D_{o_2} r(t) \end{cases} \qquad (8)$$

with $\dot{x}_e(t) = [z_1(t) \;\; \ldots \;\; z_n(t) \;\; \hat{u}(t)]^T$ where $\frac{\hat{u}(s)}{u(s)} = \frac{1}{\tau s+1}$ and

$$\begin{aligned} A_o &= A_e - \frac{B_{eu}}{\hat{\alpha}}(C_{fx} + K C_{eyx}) \\ B_{o_1} &= B_{ey} - \frac{B_{eu}}{\hat{\alpha}}(C_{fy} + K C_{eyy}) \\ B_{o_2} &= \frac{B_{eu}}{\hat{\alpha}} k_0 \\ C_o &= -\frac{1}{\hat{\alpha}}(C_{fx} + K \; C_{eyx}) \\ D_{o_1} &= -\frac{1}{\hat{\alpha}}(C_{fy} + K \; C_{eyy}) \\ D_{o_2} &= \frac{k_0}{\hat{\alpha}} \end{aligned} \qquad (9)$$

where the different matrices are defined by

$$A_e = \begin{pmatrix} -\frac{1}{\tau} & 0 & \dots & \dots & \dots & 0 & 0 \\ -\frac{1}{\tau^2} & -\frac{1}{\tau} & 0 & \dots & \dots & 0 & 0 \\ -\frac{1}{\tau^3} & -\frac{1}{\tau^2} & -\frac{1}{\tau} & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ -\frac{1}{\tau^{n-1}} & -\frac{1}{\tau^{n-2}} & \dots & \dots & -\frac{1}{\tau} & 0 & 0 \\ -\frac{1}{\tau^n} & -\frac{1}{\tau^{n-1}} & \dots & \dots & \dots & -\frac{1}{\tau} & 0 \\ 0 & 0 & \dots & \dots & \dots & 0 & -\frac{1}{\tau} \end{pmatrix}, \; C_{fx}^T = \begin{pmatrix} -\frac{1}{\tau^n} \\ -\frac{1}{\tau^{n-1}} \\ \vdots \\ -\frac{1}{\tau} \\ -\hat{\alpha} \end{pmatrix} \tag{10}$$

$$C_{fy} = \frac{1}{\tau^n}, \; B_{ey}^T = \begin{pmatrix} \frac{1}{\tau} & \frac{1}{\tau^2} & \dots & \frac{1}{\tau^n} & 0 \end{pmatrix}, \; B_{eu} = \begin{pmatrix} 0_{(1\times n)} \\ \frac{1}{\tau} \end{pmatrix}$$

$$C_{eyx} = \begin{pmatrix} 1 & 0_{(1\times n)} \\ A_{e_{((1:n-1)\times(1:n))}} & 0 \end{pmatrix}, \; C_{eyy} = \begin{pmatrix} 0 \\ B_{ey_{(1:n-1)}} \end{pmatrix}, \; C_{eu}^T = \begin{pmatrix} 0_{(1\times n)} \\ 1 \end{pmatrix}$$

One remak that the proposed approach requires less information on the system for the controller tuning : only the dynamic order, an estimation of the nonlinear functions bounded and an estimation of the input gain.

Consider the closed loop composed of the LPV system (4) and the controller/derivative pair described in *Proposition* 2. The extended presentation of the closed loop with the extended state $x(t) = [x_m(t) \;\; x_e(t)]^T$ is then given by:

$$\begin{cases} \dot{x}(t) & = & A(\mu(t))x(t) + Br(t), \\ y(t) & = & Cx(t) \end{cases} \tag{11}$$

where for all $\mu_i(t)$ verifying the convex sum property:

$$\begin{aligned} A(\mu(t)) &= \sum_{i=1}^{N} \mu_i(t) A_i \\ &= \sum_{i=1}^{N} \mu_i(t) \begin{pmatrix} A_{m_i} + B_m D_{o_1} C_m & B_m C_o \\ B_{o_1} C_m & A_o \end{pmatrix} \end{aligned} \tag{12}$$

$$A(t) = \begin{pmatrix} A_m(t) + B_m D_{o_1} C_m & B_m C_o \\ B_{o_1} C_m & A_o \end{pmatrix}, \; B = \begin{pmatrix} B_m D_{o_2} \\ B_{o_2} \end{pmatrix}, \; C^T = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ C_m^T \end{pmatrix} \tag{13}$$

For the sake of notation simplification, let us note : $A(\mu(t)) := A(t)$. The next section presents results for the stability study of this class of SISO systems under specifications.

3. Performance synthesis

This section exposes the main contribution of this paper i.e. design procedures for the stabilization of non-linear systems under specifications. Since our goal is to make the dynamic of the closed loop system (11) closed to the desired one (6), we are interested in minimizing the error $e(t)$ between the system output $y(t)$ and the ideal output $y_r(t)$ when these dynamics have the same reference $r(t)$. This will be achieved by minimizing the induced gain $\mathcal{L}_2 \to \mathcal{L}_2$ described by:

$$J = sup_{r\in\mathcal{L}_2} \frac{||e||_{\mathcal{L}_2}}{||r||_{\mathcal{L}_2}}; \ ||r||_{\mathcal{L}_2} \neq 0. \qquad (14)$$

It should be pointed out that the stability and the $\mathcal{L}_2$ gain of LPV systems is equivalent to the existence of a Lyapunov function and based on LMI optimal techniques. The result is the following:

Theorem 1. *The closed loop system composed of, the dynamic (1) with (5) and the specified dynamic (6) is globally asymptotically stable and verify $J < \Gamma$ where J is defined in (13), with a given $\Gamma > 0$, if there exists a symmetric and positive-definite matrix P ($P = P^T \succ 0$) such that the following condition holds:*

$$\begin{pmatrix} \mathcal{A}(t)^T P + P\mathcal{A}(t) + \mathcal{C}^T\mathcal{C} & P\mathcal{B} \\ \mathcal{B}^T P & -\Gamma^2 I_d \end{pmatrix} < 0, \qquad (15)$$

with:

$$\mathcal{A}(t) = \begin{pmatrix} A(t) & 0 \\ 0 & A_r \end{pmatrix}, \ \mathcal{B} = \begin{pmatrix} B \\ B_r \end{pmatrix}, \ \mathcal{C}^T = \begin{pmatrix} -C_m \\ 0_{(1\times(n+1))} \\ C_r \end{pmatrix} \qquad (16)$$

Where the different matrices are defined by (12) and (13) with (3).

Proof 1. *The dynamic of the system given by specifications (6) can be represented by the following state space by choosing $x_r(t) = [y_r(t) \ \dots \ y_r^{(n-1)}(t)]^T$:*

$$\begin{cases} \dot{x}_r(t) &= A_r x_r(t) + B_r r(t) \\ y_r(t) &= C_r x_r(t) \end{cases} \qquad (17)$$

where

$$A_r = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \dots & \dots & \dots & 1 \\ -k_0 & -k_1 & \dots & \dots & -k_{n-1} \end{pmatrix}, \quad B_r = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ k_0 \end{pmatrix}, \quad C_r^T = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix}$$

Consider now the extended state space $\mathcal{X}(t) = [x(t) \ x_r(t)]^T$ such that $x(t) = [x_m(t) \ x_e(t)]^T$ and $x_r(t)$ represent respectively the closed loop and the desired dynamic states. Thus, one gets:

$$\begin{cases} \dot{\mathcal{X}}(t) &= \mathcal{A}(t)\mathcal{X}(t) + \mathcal{B}r(t) \\ \mathcal{Y}(t) &= \mathcal{C}\mathcal{X}(t) \end{cases} \qquad (18)$$

where the different matrices are given by (16) with the appropriate matrices.

Consider the quadratic candidate Lyapunov function $V = \mathcal{X}^T P \mathcal{X}$ such that $P = P^T \succ 0$. Regarding to [53], if :

$$\dot{V} + e^T e - \Gamma^2 r^T r < 0,$$

then

$$J = sup_{r \in \mathcal{L}_2} \frac{||e||_{\mathcal{L}_2}}{||r||_{\mathcal{L}_2}} < \Gamma; \ \ ||r||_{\mathcal{L}_2} \neq 0.$$

By developing the latter condition, condition (15) is achieved and so the stability of the closed loop under the desired specifications described by the induced gain J (see [53] for the standard proof of the condition development).

Lemma 1. *The closed loop system composed of, the dynamic (1) with (5) and the specified dynamic (6) is globally asymptotically stable and verify $J < \Gamma$ where J is defined in (14), with a given $\Gamma > 0$, if there exists a symmetric and positive-definite matrix P ($P = P^T \succ 0$) such that, $\forall i = 1..N$, the following conditions hold:*

$$\begin{pmatrix} \mathcal{A}_i^T P + P\mathcal{A}_i + \mathcal{C}^T\mathcal{C} & P\mathcal{B} \\ \mathcal{B}^T P & -\Gamma^2 I_d \end{pmatrix} < 0 \qquad (19)$$

where the different matrices are described by (16).

Proof 2. *Consider the result of the previous Theorem 1. Since the matrix $A(t)$ is represented by the polytopic form (12), the condition (15) becomes:*

$$\begin{aligned} \begin{pmatrix} \sum_{i=1}^N \mu_i(t)\mathcal{A}_i^T P + P\sum_{i=1}^N \mu_i(t)\mathcal{A}_i + \mathcal{C}^T\mathcal{C} & P\mathcal{B} \\ \mathcal{B}^T P & -\Gamma^2 I_d \end{pmatrix} & < 0 \\ \Rightarrow \quad \sum_{i=1}^N \mu_i(t) \begin{pmatrix} \mathcal{A}_i^T P + P\mathcal{A}_i + \mathcal{C}^T\mathcal{C} & P\mathcal{B} \\ \mathcal{B}^T P & -\Gamma^2 I_d \end{pmatrix} & < 0 \end{aligned}$$

As $\forall i = 1..N$, $0 \leq \mu_i \leq 1$, the last condition return to the one of *Theorem 1.*

4. Simulation example: inverted pendulum

In this section, we present simulation results showing the tracking performances of the proposed restricted-model-based controller applied to an inverted pendulum system. Let us consider the polytopic representation of the inverted pendulum on a cart described by [54, 55] for a compact set of the state space:

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{gsin(x_1) - amlx_2^2 sin(2x_1)/2 - acos(x_1)u}{4l/3 - amlcos^2(x_1)} \end{cases} \qquad (20)$$

where $x_1 = \theta$ is the angle of the pendulum with respect to the vertical line, $x_2 = \dot{\theta}$ is the angular velocity,

$g = 9.8m/s^2$ is the gravity constant, $m = 2kg$ is the mass of the pendulum, $M = 8kg$ is the mass of the cart, $2l = 1m$ is the length of the pendulum, u is the control applied to the cart and $a = \frac{1}{m+M}$. The closed loop system is then described by (18) with:

$$A_{m_1} = \begin{pmatrix} 0 & 1 \\ 17.29 & 0 \end{pmatrix},\ A_{m_2} = \begin{pmatrix} 0 & 1 \\ 9.35 & 0 \end{pmatrix}.$$

Remark 2. *The inverted pendulum system is an unstable system widely used as a benchmark control problem [54, 56, 57]. The control problem of this plant has been addressed in many studies by considering for example Composite Learning Based Fuzzy Control [58], the T-S fuzzy control [59], Robust H_∞ Nonlinear Control via Fuzzy Static Output Feedback [60], Multirate Output Feedback with Discrete Time Sliding Mode Control [61] and fuzzy PID (PI, or PD) control system transformed into the fuzzy static output feedback control system [62]. However, for the stability control of this system, the methods mentioned above require the formulation of the membership functions and the establishment of the model.*

4.1. Ideal Case

The control objective is to force this system and its controller/derivative given by *Proposition 2* with an estimator parameter $\tau = 0.002$ and without loss of generality an input gain $\alpha = 1$ to follow the dynamic of the reference model (17) with

$$A_r = \begin{pmatrix} 0 & 1 \\ -k_0 & -k_1 \end{pmatrix},\ B_r = \begin{pmatrix} 0 \\ k_0 \end{pmatrix},\ C_r^T = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

where the parameters $k_0 = 25$ and $k_1 = 11$ are fixed under the desired specifications. Supposing that the specifications requires, in addition to ensuring the stability of the inverted pendulum, to guarantee an induced gain of order $J^* = 0.2$.

To verify the previous analysis results in terms of stability and performances, a simulation example of the inverted pendulum is given in Fig.1. This figure shows the regulation results of the pendulum angle θ by considering the proposed control law. From Fig.1.a, the system output successfully converges to the desired one, which means that the proposed scheme solves the regulation problem even in the case when the inverted pendulum has unknown bounded parameters and its states are not measurable. The performance of the proposed observer is shown in Fig.1.b. The proposed control law affects how fast the system output dynamic catch up with the ideal one. Finally, the control input u converges to zero, as shown in Fig.1.c. The implementation of this controller on simulation has given good results, especially in terms of the capability to control and stabilize the system rapidly.

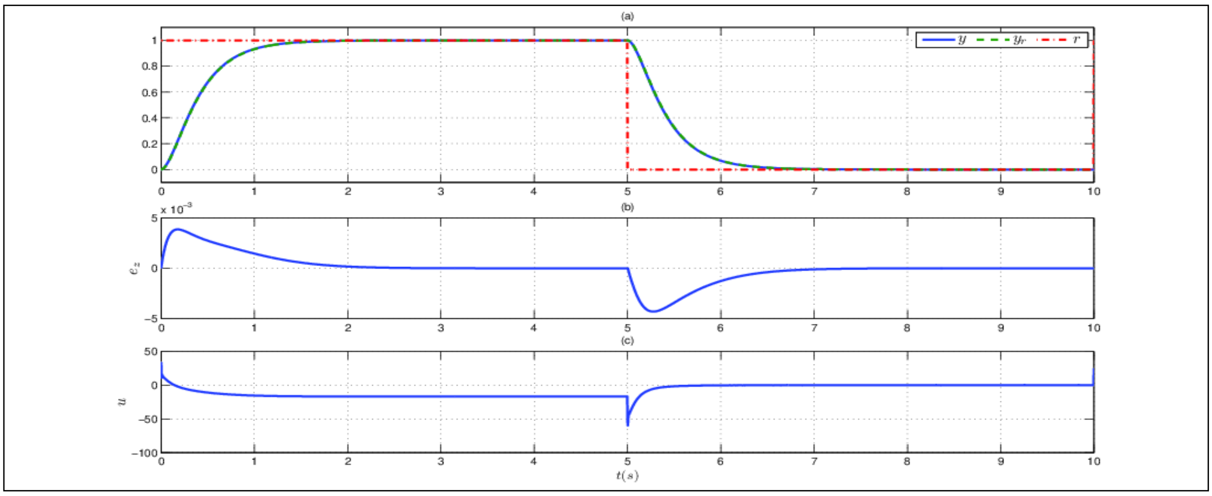

Figure 1: Response of (a) the system output dynamic $y(t) = \theta(t)$, the desired dynamic output $y_r(t)$ and the reference $r(t)$, (b) the tracking error $e_z(t) = y_r(t) - y(t)$, (c) the control input $u(t)$ of the inverted pendulum for the induced gain $J^* = 0.2$.

4.2. Influence of the noise

To illustrate the efficiency of the proposed design procedure and the attenuation problem given in this paper, we will consider the more general situation where the system is subject to perturbation $w(t)$. It is assumed that $w(t)$ is a Gaussian noise with a variance 100 (see Fig.2.) influences on the angular velocity dynamic. The desired attenuation level (the induced gain) is $J^* = 0.2$.

From Fig.3. it can be seen from this results that in the case of the existence of the noise on the system state, the proposed controller scheme greatly converges to the desired dynamic under the desired specifications (see Fig.3.a). It can be seen from Fig.3.b. that the tracking error converges to zero quickly. Yet, Fig.3.b. shows that the control signal u becomes affected by the chattering phenomena.

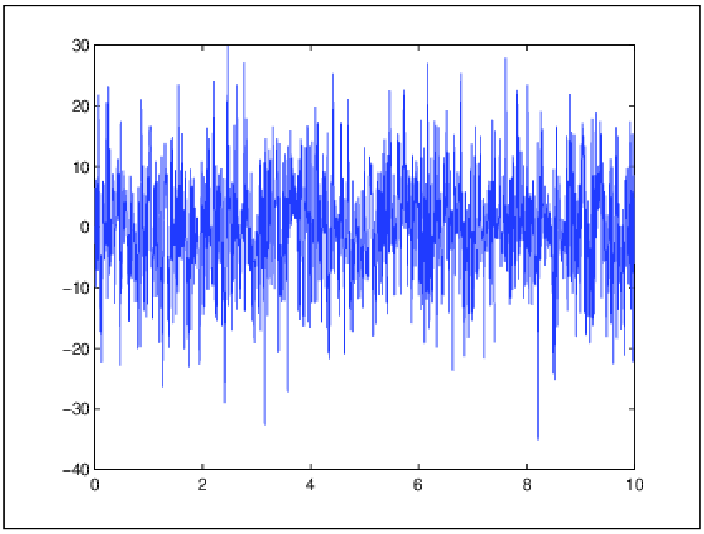

Figure 2: Gaussian noise signal.

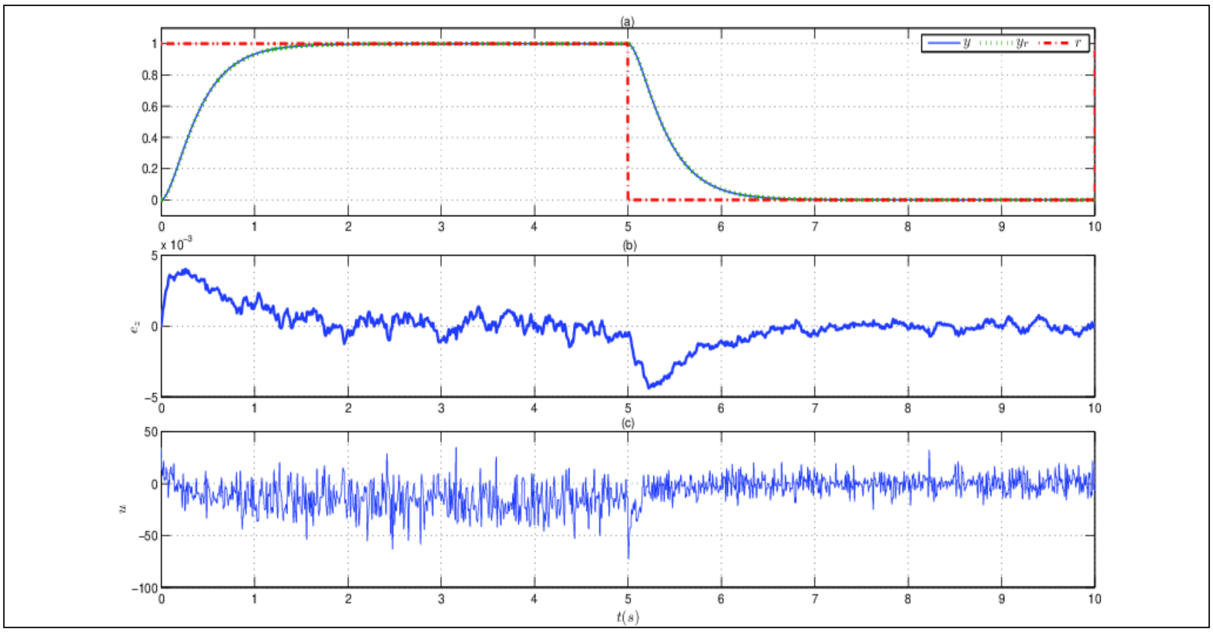

Figure 3: Response of (a) the system output dynamic $y(t) = \theta(t)$, the desired dynamic output $y_r(t)$ and the reference $r(t)$, (b) the tracking error error $e_z(t) = y_r(t) - y(t)$, (c) the control input $u(t)$ of the disturbed inverted pendulum for the induced gain $J^* = 0.2$.

5. Conclusion

The problem of performance analysis of non-linear systems described by polytopic model with unmeasurable bounded parameter variables has been investigated. Considering the model free control developed in previous works, a simplified design of the controller was proposed. The controller design did not use an explicit model or the structural information of the plant. Favourable asymptotic convergence and improved tracking performances could be achieved through the induced gain $\mathcal{L}_2$ study. As demonstrated in the simulation examples, the proposed method was provide its effectiveness. Yet, it still limited on the case of non-linear SISO system with bounded parameter and with no invariant zero. In addition, the proposed control law still based on a model and the setting of this controller under the desired specifications requires some knowledge about the system. Yet, one notes that it is possible to empirically adjust this controller if the system is not critical and/or stable using as initial value of a value less than the system dynamics (index value measured by test).

The proposed approach requires less information on the system for the controller tuning : only the dynamic order, an estimation of the nonlinear functions bounded and an estimation of the input gain. This work has opened new perspectives of study: proposing a simple tuning method for the controller design involving lesser calculation efforts and could be implemented much easier than the traditional controllers. Another interresting point is to study the influence of fast dynamics (i.e. consider a controller order lower than the system) or extending the particular case (Multi-Input Multi Output, zero dynamics, …).

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